LAMB WAVE ASSESSMENT OF STIFFNESS DEGRADATION IN FATIGUED COMPOSITES

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INTRODUCTION

The introduction of advanced composites into the next generation of aerospace structures will require these materials to operate under severe mechanical loading conditions for thousands of hours. Degradation with fatigue can cause a significant decrease in the load-carrying capability of these materials which could compromise safety. Therefore, techniques are required to nondestructively evaluate the integrity of composites which will be subjected to mechanical fatigue. Among the various techniques available, Lamb waves offer a convenient method of evaluating these composite materials.

Studies have been conducted which show a reduction in Lamb wave velocity due to a loss of stiffness caused by matrix cracking [1-4]. Karim, et al. [5] and Mal, et al. [6] have used inversion techniques to ascertain the material parameters of composites from experimental Lamb wave data. Recently, Shih, et al. [7] used measurements of the extensional plate mode velocity to calculate laminate stiffness constants in fatigued composites. In this study, a Lamb wave scanning system has been used to measure the flexural plate wave velocity over a wide frequency range for mechanically fatigued composites. From the flexural mode velocity data, the bending and out-of-plane stiffness values were determined for each of the specimens. The results were compared to crack density measurements as well as the in-plane stiffness which was derived from velocity measurements of the extensional mode. The effectiveness of the scanning system in measuring fatigue damage is then discussed.

LAMINATED PLATE THEORY

For a unidirectional composite lamina with the x-axis defined as along fibers, the y-axis transverse to the fibers, and the z-axis out of the plane of the plate, the stress-strain relationship is given by

\[ \{\sigma_i\} = [Q_{ij}][\varepsilon_j] \]  \hspace{1cm} (1)
where the $Q_{ij}$ are the stiffness coefficients, $\sigma$ represents the stresses, and $\varepsilon$ represents the strains. The in-plane stiffness, $A_{11}$, bending stiffness, $D_{11}$, and the out-of-plane stiffness, $A_{55}$, for the entire laminate are obtained by integrating the $Q_{ij}$ through the thickness of the plate. These stiffness values are defined as [8]

$$
(A_{11}, D_{11}) = \frac{h}{2} \int_{-h/2}^{h/2} (Q_{11})' k(1, z^2) dz
$$

$$
A_{55} = k_5^2 \frac{h}{2} \int_{-h/2}^{h/2} (Q_{55})' k dz
$$

(2)

where the subscript $k$ represents each layer in the laminate, $k_5$ is a shear correction factor, and $h$ is the total thickness of the plate. The $Q_{ij}'$ are the transformed stiffness components which have been rotated according to the orientation of each ply with respect to the wave propagation direction. Relationships between the $Q_{ij}$ and the elastic stiffness constants, $c_{ij}$, can be found in Jang [9] and equations relating the engineering constants ($E_1, E_2, G_{12}, v_{12}$, and $v_{21}$) to the $Q_{ij}$ can be found in Daniel and Ishai [10].

PLATE WAVES

For a wave travelling in the $0^\circ$ direction of a symmetric crossply laminate, the velocity of the extensional mode, $v_s$, is related to the in-plane stiffness, $A_{11}$, by [11]

$$
v_s = \sqrt{\frac{A_{11}}{\rho h}}
$$

(3)

where $\rho$ is the density and $h$ is the plate thickness. Thus, if the density and plate thickness are known, the in-plane stiffness can be obtained from a measurement of the extensional mode velocity. For flexural wave propagation in the $0^\circ$ direction of a symmetric crossply laminate, the dispersion relation is given by [8]

$$
(D_{11}k^2 + A_{55} - I\omega^2)(A_{55}k^2 - \rho^* \omega^2) - A_{55}^2 k^2 = 0
$$

(4)

where $k$ is the wavenumber, $\omega$ is the angular frequency, and $I$ and $\rho^*$ are defined as [8]

$$
(\rho^*, I) = \frac{h}{2} \int_{-h/2}^{h/2} \rho(1, z^2) dz
$$

(5)

If the density and thickness are known, the stiffnesses can be obtained by adjusting the constants in Eq. (4) in order to produce the best non-linear least squares fit to experimentally obtained values of $w$ and $k$. For a further description of plate theory and how the laminate stiffnesses relate to the flexural dispersion curve, the reader is referred to Tang, et al. [8].

The effects of altering the values of $A_{55}$ and $D_{11}$ in Eq. (4) on the flexural dispersion curve are shown in Fig. 1. As can be seen in the figure, decreasing $D_{11}$ by 25% does not
significantly change the curve. However, a larger shift in the curve is seen when $A_{55}$ is reduced by 25%. Shown in Fig. 2 is a plot of the percent reduction in velocity as a function of frequency for changes in each stiffness value. The figure clearly shows that reducing $D_{11}$ has a greater effect on the velocity at lower frequencies and reductions in $A_{55}$ alter the velocity.
dispersion curve at higher frequencies. Therefore, the behavior of the dispersion curve is dominated by \( D_{11} \) at low frequencies and by \( A_{55} \) at higher frequencies.

**STIFFNESS MEASUREMENT TECHNIQUE**

In this measurement, two transducers are used in a pulse/receive arrangement to determine the velocity of the flexural plate mode over a wide frequency range. The receiving sensor is moved by small increments in order to assure that the same peak in the waveform is followed over the total separation distance. The velocity at each frequency is calculated from the known transducer separation and the measured time-of-flight. Once the velocity as a function of frequency is measured, the bending and out-of-plane shear stiffnesses are obtained by adjusting the values of \( D_{11} \) and \( A_{55} \) in Eq. (4) in order to produce the best fit to the experimental data. The velocity measurements are accurate and repeatable to within 1\% resulting in reconstructed stiffness values repeatable to within 4\% [12]. A mechanical scanner is used to move the sensors over the surface to map the time-of-flight, velocity, and stiffness of the entire specimen. Access to only one side of the material is required and no couplants are required because the sensors are dry coupled to the surface of the plate.

**SCANNER ACCURACY EVALUATION**

To evaluate the accuracy of the stiffness measurements obtained from Lamb wave velocity measurements, the elastic stiffness was measured in 25 different regions on a large rolled aluminum plate. The plate had dimensions of 50.8 cm by 38.1 cm and a thickness of 0.32 cm. The scan area was 12.0 cm by 12.0 cm with a step size of 3.0 cm and the starting point was 15.0 cm away from the two closest edges of the plate. For the velocity measurement at each location, the sensor separation was varied from 2.75 cm to 4.75 cm in increments of 0.5 cm. An 8-cycle Gaussian sine wave was used to generate the signal and the sampling rate was chosen to be 25 MHz. The frequency was swept from 30 to 250 kHz in 10-kHz steps and the velocity at each frequency was measured. Since the material is isotropic, only two independent elastic constants exist. From the data, the dispersion curve was reconstructed and values for the longitudinal and shear modulus which best fit the experimental data were obtained.

Longitudinal and shear modulus values of 67.7 GPa and 25.0 GPa [13], respectively, were used in the laminated plate theory model to obtain a theoretical dispersion curve to compare to the experimental data and reconstructed dispersion curve. Shown in Fig. 3 is the experimental velocity data, the reconstructed dispersion curve, and theoretical dispersion curve. The standard deviations in the velocity at each frequency for the 25 measurement locations were all within 1\% of the average with most values being within 0.5\% from the average. This small deviation produces error bars on the order of the size of the symbols in the figure. The average reconstructed stiffnesses and the standard deviations for the 25 measurement locations were 70.0 ± 1.5 GPa for the longitudinal modulus and 26.3 ± 1.7 GPa for the shear modulus. Thus, the average experimentally obtained longitudinal and shear modulus values were 3.4\% and 5.2\%, respectively, from the theoretical values found in the literature.

**COMPOSITE SAMPLES AND FATIGUE PROCESS**

The composite samples studied were AS4/3501-6 graphite/epoxy with a stacking sequence of [0/90]_S. The eleven coupons were 3.8 cm by 27.9 cm and had a nominal thickness of 0.12 cm. Before the fatiguing process, four samples had two 0.635-cm strain
gages attached: one axial and one transverse. These specimens were then loaded quasi-
statically to failure. From the measured stress, axial strain, and transverse strain, values
were calculated for Young’s modulus, Poisson’s ratio, and ultimate strength. The ultimate
strength was defined as the stress at which the samples failed.

Five of the remaining specimens were subjected to tension-tension fatigue in a 244-
kN (55 kip) capacity load frame. They were fatigued for $10^5$ cycles at a frequency of 10 Hz
and at an R value (minimum load/maximum load) of 0.35. The upper stress level had a
value of 200 MPa, which was 45% of the ultimate strength. To measure the crack density,
the number of cracks occurring in a 2.5-cm section of the plate was counted by examining
the polished edges of the coupons under a microscope. Two of the specimens were left
unfatigued as control samples.

STIFFNESS MEASUREMENTS FOR FATIGUED COMPOSITES

To evaluate the effects of fatigue on composite stiffness, the scanning system was
used to measure time of flight, velocity, and stiffness on two unfatigued and five fatigued
specimens. Due to the size of the samples, only a small number of measurements could be
obtained. Stiffness measurements were made in four regions near the middle of the plate for
each sample. The sensor separation was varied from 3.75 cm to 5.75 cm in increments of
0.4 cm. A 4-cycle Gaussian-enveloped sine wave was used to generate the signal and the
sampling rate was 25 MHz. The frequency was swept from 50 kHz to 250 kHz in 10-kHz
steps and the velocity at each frequency was obtained. The thickness was given above and
the density, estimated from common values for composites, was taken to be 1580 kg/m$^3$.
From the velocity measurements for each sample, the dispersion curve was reconstructed
using values for $A_{55}$ and $D_{11}$ in Eq. (4) which produced the best fit to the experimental data.
Shown in Fig. 4 are the average experimental velocity measurements for an unfatigued sample and a fatigued sample. Also shown are the reconstructed dispersion curves for each specimen. The error bars represent the standard deviation in velocity at each frequency for the four measurement regions. As can be seen from the figure, the dispersion curve for the fatigued sample is clearly shifted from that of the unfatigued sample. The four stiffness measurements for each sample were averaged, resulting in an estimate of the out-of-plane stiffness, $A_{55}$, of $(3.99 \pm 0.03) \times 10^6$ N/m for the unfatigued sample and $(3.21 \pm 0.24) \times 10^6$ N/m for the fatigued sample. The values obtained from the reconstruction for $D_{11}$ were $12.37 \pm 0.64$ N•m for the unfatigued sample and $13.00 \pm 2.60$ N•m for the fatigued sample. Both values of stiffness for the fatigued sample changed from the unfatigued specimen. The value of $A_{55}$ decreased by nearly 20% while $D_{11}$ increased by 5%.

It is expected, and previous strain gage measurements show [1], that matrix cracking due to fatigue damage leads to a decrease in elastic moduli. Thus, the decrease observed in $A_{55}$ was expected. However, the small increase in $D_{11}$ was not anticipated. The large standard deviations associated with the average values of $D_{11}$ suggest that the measured values are not particularly reliable and this may be the reason for the apparent increase in $D_{11}$. The reason for the large scatter in the measurements is most likely due to the insensitivity of the dispersion curve to changes in $D_{11}$ over the measurement frequency range (50 kHz to 250 kHz). In this region, the parameter dominating the behavior of the curve will be $A_{55}$ (see Fig. 2). Only a few data points exist at the very low frequencies where $D_{11}$ controls the behavior of the curve. Due to this lack of data, the constant $D_{11}$ will be inaccurate, which is reflected in the large standard deviations. Therefore, the more accurate constant is the out-of-plane stiffness, $A_{55}$.

In addition to the flexural mode velocity measurements presented here, the velocity of the extensional mode for this set of samples has also been measured in a previous study [1]. From these measurements, the in-plane stiffness, $A_{11}$, was calculated from the known
density and thickness using Eq. (3). The values for both $A_{11}$ and $A_{55}$ were normalized with respect to their value for the unfatigued (0 crack density) specimens. The normalized $A_{11}$ and $A_{55}$ values are plotted as a function of the measured crack density in Fig. 5. Both of the normalized stiffnesses decrease as a function of crack density, but the in-plane stiffness did not change as much as the out-of-plane stiffness. This difference can be explained by considering the effects of the composite constituent properties on the different elastic moduli. The constant $A_{11}$ is dominated by the 0° fibers and thus, less affected by the matrix properties such as fatigue induced matrix cracking. The stiffness $A_{55}$, however, is controlled heavily by the matrix since the out-of-plane shear carrying capabilities of the composite are matrix dominated. Therefore, matrix cracking has a greater effect on $A_{55}$ than $A_{11}$, which is the trend shown clearly in Fig. 5.

CONCLUSION

The Lamb Wave Imager™ is a new tool for nondestructively measuring the elastic properties of a composite material. The scanner requires access to only one side of a specimen with no immersion or couplants. It has been shown by this study to be a very effective method in providing a quantitative measure of fatigue damage in composite materials. The out-of-plane stiffness measurements correlated well with crack density data and showed a similar trend as measurements of the in-plane stiffness.

The scanning system provides a fast and accurate method of nondestructively obtaining quantitative information about materials. Such a measurement would also be sensitive to stiffness changes caused by impact damage, delaminations, fatigue, and debonds. It should also detect manufacturing anomalies such as inconsistent thickness, fiber misalignment, porosity, and low fiber volume fraction. This instrument provides new capabilities to process engineers and material engineers that allow them to inexpensively map out material properties with higher resolution than before. Such a capability is

![Figure 5](image-url)  
Figure 5. Plot of the normalized out-of-plane stiffness, $A_{55}$, and the normalized in-plane stiffness, $A_{11}$, as a function of crack density.
particularly useful when there are variations in material properties across a given specimen, such as might be the case for localized damage or variability caused by the manufacturing process.

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