CALIBRATION OF A MAGNETIC FIELD PROBE AS A DIAGNOSTIC FOR
LASER-PRODUCED PLASMA

An Honors Thesis submitted by

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I. INTRODUCTION

This report focuses on work done by the author to improve the construction and calibration of magnetic field probes designed to measure fast plasma phenomena, such as that produced by laser ablation of a solid target. Small, high-frequency inductive magnetic field probes were constructed and calibrated using a network analyzer and oscilloscopes. Interference, common for such magnetic probes, was observed during calibration. In this case it was traced to a lack of impedance matching between the signal cables. Methods for reducing this interference were formulated and tested. A prototype analog integrator was built and tested in hopes of removing discrete integration error from the collection of magnetic field data. Finally, a simple laser-plasma experiment was conducted using the probes. This experiment revealed that the probes can make meaningful measurements in a real laboratory setting. The sources of error in the data were examined, and the plasma magnetic field measurements were compared to prior works and found to agree.

II. INTRODUCTION TO PLASMA

Plasma is an important part of our world. All stars, including our sun, are composed of plasma, so understanding plasma is important for understanding space physics. Lightning and flames are also examples of plasma. Plasma has many applications: fluorescent lighting relies on plasma, as do plasma television screens. Plasma torches are used in welding and other industrial operations. Plasma is also used in materials processing for many applications from jet engine parts to computer chips. Plasma may one day provide a nearly limitless source of energy in the form of nuclear fusion. This possibility drives much of the current research in plasma physics.

Plasma is the fourth state of matter, after solid, liquid and gas. Just as solids change to liquids and liquids to gases as the temperature increases, gases change to plasma at high temperature. At high temperature, the atoms of the gas are moving very fast, and collisions between atoms knock electrons away from their nuclei. Thus plasma are composed of charged atoms (called ions) mixed with free electrons. The separation of electrons and ions allows plasma, unlike gas, to conduct electricity. This also means that plasma can create, and respond to, electric and magnetic fields. This makes plasma behave in much
more complicated ways than does gas.

A. Laser-Produced Plasma

Laser-produced plasma (hereafter laser plasma) has been of interest in at least two important areas of physics research. Most prominent is laser plasma research involved in the inertial confinement method of fusion energy production. Inertial confinement is one of the most promising approaches to harnessing nuclear fusion power to replace the use of fossil fuels and nuclear fission power. In this approach, laser beams are fired at a small spherical shell of frozen hydrogen gas. Some of the hydrogen is vaporized and ionized into a plasma, and expands rapidly away from the target. However, by Newton’s third law, the outward acceleration of the plasma causes an inward force which compresses and heats the hydrogen until it ignites a nuclear fusion reaction, releasing energy. Investigation of laser plasma revealed that it contained self-generated (or spontaneous) magnetic field. It has been feared that this field might disrupt efforts to achieve effective ignition of the fusion reaction.\textsuperscript{12,17,23}

The magnetic field of laser plasma is also of interest to current efforts to recreate astrophysical plasma effects in the laboratory. Specifically, laser plasma can be used to drive collisionless shocks into preexisting magnetized plasma, mimicking planetary bow shocks, coronal mass ejections, and supernovae. Laboratory experiments can help to verify existing theoretical and computer models of these phenomena, since it is often impossible to make measurements on actual space plasma. Additionally, it has been postulated that very strong electric fields arising in collisionless shocks may be responsible for accelerating charged particles to ultra-relativistic velocities; it is hoped that such acceleration might be experimentally observed.\textsuperscript{6}

B. Spontaneous Magnetic Field Generation

It is generally agreed that the presence of non-parallel temperature and density gradients are the primary source of the magnetic field observed far from the laser burn spot, although other possible sources such as the dynamo effect, thermoelectric effect, and laser interactions are mentioned.\textsuperscript{3,4,10,15,21,23} How these gradients produce a magnetic field may be seen by deriving the equation governing the magnetic field from the generalized Ohm’s Law\textsuperscript{5} for a
simplified single-fluid plasma model,

\[ E + v_e \times B = \eta j + \frac{1}{en_e} (j \times B - \nabla p_e) \]  

(1)

where \( E \) is the electric field, \( v_e \) is the plasma electron velocity, \( B \) is the magnetic field, \( j \) is the current density, \( \eta \) is the plasma resistivity, \( e \) is the elementary charge, \( n_e \) is the plasma electron density, and \( p_e \) is the electron pressure. This equation relates the basic forces on the electrons in the single-fluid plasma model: electric and magnetic fields, electrical resistance, and pressure. The Hall term, \( j \times B \), is usually neglected in treating laser plasma since it has little influence.\(^4\) (See Griffiths,\(^9\) or any text on vector analysis for more information on vector operations.)

Following Parlar,\(^{15}\) this equation can be rewritten to give the rate of change of the magnetic field. First, Eq. 1, without the Hall term, can be solved for \( E \),

\[ E = -v_e \times B + \eta j + \frac{1}{en_e} \nabla p \]  

(2)

and the curl of both sides can be taken,

\[ \nabla \times E = \nabla \times (-v_e \times B) + \nabla \times (\eta j) - \nabla \times \left( \frac{1}{en_e} \nabla p_e \right). \]  

(3)

Using Faraday’s Law in differential form,\(^9\) \( \nabla \times E = -\frac{\partial B}{\partial t} \), and distributing we have,

\[ \frac{\partial B}{\partial t} = \nabla \times (v_e \times B) - \eta \nabla \times j + \nabla \times \left( \frac{1}{en_e} \nabla p_e \right). \]  

(4)

Since we neglect the displacement current,\(^{15}\) we have Ampere’s Law\(^9\) as \( \nabla \times B = j \), so we can rewrite the second term from the right-hand side of Eq. 4 as,

\[ -\eta \nabla \times j = -\eta \nabla \times (\nabla \times B) \]  

(5)

\[ = -\eta [\nabla(\nabla \cdot B) - \nabla^2 B] \]  

(6)

\[ = \eta [-\nabla(0) + \nabla^2 B] \]  

(7)

\[ = \eta \nabla^2 B, \]  

(8)

where we have also used the vector identity for the curl squared, together with \( \nabla \cdot B = 0 \) from Maxwell’s equations.\(^9\) We can similarly rewrite the third term using the thermodynamic definition of pressure as \( p_e = n_e k T_e \) where \( k \) is the Boltzmann constant and \( T_e \) is the electron temperature.\(^{15}\) Here we use of the product rule in vector notation,

\[ \nabla(AB) = A\nabla B + B\nabla A \]
and the formula for the cross-product of a scalar times a vector,

\[ \nabla \times A \mathbf{V} = A \nabla \times \mathbf{V} + \nabla A \times \mathbf{V} \] (9)

where \( A \) and \( B \) are scalar functions and \( V \) is a vector function. Applying these yields,

\[ \nabla \times \left( \frac{1}{e n_e} \nabla p \right) = \nabla \times \frac{1}{e n_e} \nabla (n_e kT_e) \] (10)

\[ = \nabla \times \frac{1}{e n_e} \left[ n_e k \nabla T_e + T_e k \nabla n_e \right] \] (11)

\[ = \nabla \times \left[ \frac{k}{e} \nabla T_e + \frac{k T_e}{e n_e} \nabla n_e \right] \] (12)

\[ = \frac{k}{e} \nabla \times \nabla T_e + \nabla \times \left( \frac{k T_e}{e n_e} \nabla n_e \right). \] (13)

This result can be further simplified by using the identity, \( \nabla \times \nabla A = 0 \). Applying this identity twice and using Eq. 9 again, we have,

\[ \nabla \times \left( \frac{1}{e n_e} \nabla p \right) = \nabla \times \frac{k T_e}{e} \left( \frac{\nabla n_e}{n_e} \right) \] (14)

\[ = \frac{k T_e}{e} \nabla \times \frac{\nabla n_e}{n_e} + \nabla \frac{k T_e}{e} \times \frac{\nabla n_e}{n_e} \] (15)

\[ = \frac{k}{e} \nabla T_e \times \frac{\nabla n_e}{n_e}. \] (16)

Finally, the modified terms can be substituted into Eq. 4 to yield the governing equation for the magnetic field,

\[ \frac{\partial B}{\partial t} = \nabla \times (v_e \times B) + \eta \nabla^2 B + \frac{k}{e} \nabla T_e \times \frac{\nabla n_e}{n_e}. \] (17)

This term indicates that non-parallel gradients in temperature and density can produce magnetic fields. The second term on the right hand side describes the diffusion of the magnetic field, while the first term describes field convection and is also related to the dynamo effect. Convection refers to the magnetic field being dragged along by the plasma, where diffusion means that the field passes through the plasma.

III. INTRODUCTION TO MAGNETIC FIELD PROBES

In order to understand and use plasma, it is necessary to measure its properties. These properties include its temperature, pressure, voltage, current and magnetic field, among many others. Measuring plasma properties can be difficult. Often the plasma is so hot that
no instrument can be inserted into it, and in other cases instruments can disrupt the plasma properties being measured. Consequently, measurements are often made by passing laser or particle beams through the plasma, or by collecting radiation or particles from the plasma itself.

The magnetic field is an important property of laser plasmas as we have seen above. There are various ways to detect the magnetic field of a plasma, but the most straightforward diagnostic is a simple loop of wire. Faraday’s Law in integral form states that a changing magnetic flux through a closed curve in space creates an electric field around this curve,

$$V = -\frac{d\Phi_B}{dt}$$  \hspace{1cm} (18)

where $\Phi_B$ is the magnetic flux through the curve $V$ is the electromotive force, or voltage. If we let the curve be a coil of wire with $N$ turns and area $a$, and assume a constant magnetic field $B_\perp$ perpendicular to the plane of the coil, this becomes,

$$V = -\frac{d}{dt}(aNB_\perp)$$  \hspace{1cm} (19)

$$= -aN\frac{dB_\perp}{dt}$$  \hspace{1cm} (20)

By measuring this resulting voltage across the leads of the coil, the time-derivative of the magnetic field within the coil can be determined. A B-dot probe is a plasma diagnostic instrument that takes advantage of this property. A B-dot probe usually consists of a single- or multi-turn coil mounted on or enclosed in a support that allows the coil to be inserted into or near the plasma. The term ‘B-dot’ arises from the mathematical notation $\frac{dB}{dt} = \dot{B}$. The magnetic field itself can be determined by integrating the B-dot signal in real-time using an analog integrator circuit or numerically after digitization of the signal. 

For several reasons, B-dot probes must be calibrated to achieve accurate field measurements. Since probes tend to be small, physical measurements of the coil area are difficult and inaccurate. In addition, non-ideal electronic effects, such as self-inductance in the coil or stray inductances and capacitances in the circuit, often occur. Calibration is often accomplished by measuring the probe’s response to a known sinusoidal magnetic field from a Helmholtz coil. By taking measurements at multiple frequencies, the probe’s frequency response can be determined. A network analyzer can be used, which automates the collection of data from many frequencies and allows the phase of the probe’s response to be determined as well.
A. Probe Construction

For this report, probes were built using identical construction techniques to those described by Everson et al.\(^8\) The probe consists of two paired segments of magnet wire wrapped 5 times around each axis of a 3-axis heat-resistant plastic core, yielding coils of approximately 1 \(\text{mm}\) diameter. The core is placed at the tip of an alumina shaft and covered by a glass capillary tube. The leads from all six loops are twisted into a bundle that runs the 8-inch length of the hollow shaft, terminating in connections to six RG-178 coaxial cables. The coaxial cables carry the signal through the 3/8-inch stainless steel vacuum shaft and terminate in LEMO connectors (see Fig. 1).

![FIG. 1: Illustration of a probe, showing the wiring for a single axis.](image)

The two loops on each axis are connected to their respective cables in the opposite sense so that a changing magnetic flux will produce equal but opposite voltages in the two loops (Fig. 1). Any capacitive pickup (that is, voltage produced on the probe by the plasma through electrostatic induction) will have the same polarity on each loop. Therefore, when the signals from the two loops are subtracted by the differential amplifier, the common capacitive voltage will cancel, leaving the combined magnetically induced signal. Letting \(V_{\text{meas},1}\) and \(V_{\text{meas},2}\) represent the signals in the opposite windings, \(V_B\) the voltage induced by the changing magnetic flux, and \(V_E\) the capacitive (electrostatic) signal, it is easy to see how the differential amplifier eliminates the electrostatic component of the signal:

\[
V_{\text{meas},1} - V_{\text{meas},2} = (V_B + V_E) - (-V_B + V_E) = 2V_B.
\] \hspace{1cm} (21)

B. Introduction to Circuit Theory

A brief introduction to elementary methods of circuit theory is a necessary preface to the discussion of B-dot probe calibration. The focus is on the use of frequency-domain analysis.
This introduction follows the development presented in Scherz. Many circuit equations are second-order or higher differential equations, which can be difficult or impossible to solve explicitly in the time-domain. The frequency-domain method assumes the voltages and currents of a circuit are sinusoids, with the properties of amplitude, frequency, and relative phase. This is often a good approximation: many circuits involve sinusoidal or periodic functions. Additionally, by Dirichlet’s Theorem, any continuous function of finite duration in time can be expressed mathematically as the sum or integral of sinusoidal functions. This means that using Fourier Transforms, a time-domain function can be decomposed into components of varying frequency, and the results of the frequency-domain analysis can be applied. Thus despite the assumption of sinusoidal signals, the results are generally applicable.

It is customary to define a complex function for a voltage or current and specify that the real component of the complex function represents the measured voltage. We write,

\[ V(\omega) = V_0 e^{i(\omega t + \phi)} \]  

(22)

where the real numbers \( V_0, \omega \) and \( \phi \) represent the amplitude, frequency and relative phase, respectively, of the input voltage function. Here \( i \) is the unit imaginary number. The Euler relation,

\[ e^{i\theta} = \cos(\theta) + i\sin(\theta). \]  

(23)

can be applied to show that the real component of the complex voltage function is a simple sinusoid,

\[ \text{Re}|V(\omega)| = V_0 \cos(\omega t + \phi). \]

This notation allows capacitance and inductance to be treated as complex resistances. The defining equation of a capacitor is \( Q = VC \), where \( Q \) is the charge on the capacitor and \( C \) is its capacitance. Differentiating both sides with respect to time we have:

\[ I = \frac{dQ}{dt} = C \frac{dV}{dt}. \]

Inserting \( V(\omega) \) we have,

\[ I(\omega) = C \frac{d}{dt} \left( V_0 e^{i(\omega t + \phi)} \right) \]

(24)

\[ = CV_0 (i\omega) e^{i(\omega t + \phi)} \]  

(25)

\[ = [i\omega C] V(\omega) \]  

(26)
Then if we define the quantity $Z_c$, called the capacitive reactance, to be,

$$Z_c = \frac{1}{i\omega C}$$

we see that this quantity plays a role similar to that of resistance in Ohm’s Law,

$$V(\omega) = I(\omega)Z_c(\omega).$$

Similarly, inductors posses inductive reactance, defined $Z_l(\omega) = i\omega L$. The sum of the resistances and reactances of a circuit element is known as the element’s impedance: $Z(\omega) = R + Z_c(\omega) + Z_l(\omega)$. The impedance obeys the complex form of Ohm’s Law,

$$V(\omega) = I(\omega)Z(\omega). \quad (27)$$

Notice that reactances vary with frequency, whereas resistance does not. Also notice that reactances change the phase relationship between the voltage and the current. Using Euler’s relation (Eq.23), we can write the impedance in polar form as

$$Z(\omega) = |Z(\omega)|e^{i\theta(\omega)}. \quad (28)$$

Here $|Z(\omega)|$, the magnitude of the impedance, and $\theta(\omega)$, the argument of the impedance, are defined by

$$|Z(\omega)| = \sqrt{R^2 + (R_c(\omega) + R_l(\omega))^2}, \quad (29)$$

$$\theta(\omega) = \tan^{-1}\frac{Z_l(\omega) + Z_c(\omega)}{R}. \quad (30)$$

The magnitude of the total impedance relates the magnitudes of the voltage and current, while the argument of the impedance relates the phase shift between them:

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \quad (31)$$

$$|Z(\omega)|e^{i\theta(\omega)} = \frac{V_0e^{i(\omega t+\phi_V)}}{I_0e^{i(\omega t+\phi_I)}} = \frac{V_0}{I_0}e^{i(\phi_V-\phi_I)}. \quad (32)$$

Equating the magnitudes and arguments of the left- and right-hand sides gives:

$$|Z(\omega)| = \frac{V_0}{I_0}, \quad (34)$$

$$\theta(\omega) = \phi_V - \phi_I. \quad (35)$$
This shows that the magnitude of the impedance determines the ratio between the amplitude of the current and voltage, while the argument determines the phase shift between the current and voltage. In a similar fashion, the input and output voltage functions of a circuit can be related by a complex function of frequency, called the gain, \( g(\omega) \):
\[
\frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)} = g(\omega).
\]

(C. Existing Probe Calibration Method)

The B-dot calibration was also conducted following Everson et al\(^8\), with the omission of the probe testing via an oscillating test field. The calibration electronics were assembled as shown in Fig. 2. The network analyzer (Agilent/HP E5100, 10 kHz - 180 MHz) was set to measure the real and imaginary components of \(|\frac{V_{\text{meas}}}{V_{\text{ref}}}(\omega)|\). Here \( V_{\text{ref}}(\omega) \) is proportional to the current through the magnetic field generator and, hence, proportional to the magnetic field. The ratio \( \frac{V_{\text{meas}}}{V_{\text{ref}}}(\omega) \) represents the gain function of the probe/field-generator system.

The circuit of the probe tip was modeled as shown in Fig. 3. Here \( C_s \) is the capacitance.
between the twisted leads, $r$ is the resistance of the coils and leads, and $R_s$ is the terminating load (50 Ω). A pair of equations were derived which, when fitted to the gain and phase, yield two calibration factors: the effective area per turn, $a$, and the coil’s self-inductance $\tau_s$ (see Eqs. 3,4). A low-frequency calibration using an 11-cm Helmholtz coil was used to find $aN$ via curve fitting the imaginary part of the following equation (Eq. 37) to the frequency data. Here $g$ is the amplifier gain and $R_p$ is the resistance across which $V_{ref}$ is measured in the Helmholtz circuit. Note that $\tau_s$ cannot be distinguished from $\tau$, the cable delay, in the low frequency case.

$$\frac{V_{meas}(\omega)}{V_{ref}(\omega)} = aNg\left(\frac{16}{5\sqrt{2}}\right)\frac{\mu_0}{rR_p} \left[(\tau_s + \tau)\omega^2 + i\omega\right]$$

(37)

The network analyzer did not supply sufficient power to drive the Helmholtz coil at high frequency (above 1 $MHz$), so a straight wire segment was used as the field generator to collect high frequency data. This data was fitted to a more detailed equation to determine $\tau$ and $\tau_s$:

$$\frac{V_{meas}(\omega)}{V_{ref}(\omega)} = \frac{aN\alpha\mu_0}{\pi r R_p} \frac{\omega}{(\omega \tau_s)^2} \left\{\omega \tau_s \cos(\omega \tau) - \sin(\omega \tau)\right\} \\
+ i \left[\omega \tau_s \sin(\omega \tau) + \cos(\omega \tau)\right]\}$$

(38)

When analyzing experimental data, $\tau$, the electrical time delay between the probe and the magnetic field generator circuitry, is ignored. However, $\tau_s$ is used as a first-order correction to the classical B-dot theory as follows:

$$\frac{V_{meas}(\omega)}{V_{ref}(\omega)} = aNg\left(\frac{16}{5\sqrt{2}}\right)\frac{\mu_0}{rR_p} \left[\tau_s \omega^2 + i\omega\right].$$

(39)

The data analysis process was automated by an in-house software routine that also integrated the signal using the trapezoidal-sum method to solve for the corrected magnetic field as a function of time (Eq. 40).

$$B(t) = \frac{1}{aN} \left[ \int V_{meas}(t) dt + \tau_s V_{meas}(t) \right]$$

(40)

D. Motivation for Present Work

It was noted that the upper frequency limit for which the high frequency model fit the calibration data was approximately 40 $MHz$, due to interference. This interference had
been ascribed to pickup of the electromagnetic radiation from the test field by components of the probe other than the coils. The interference occurred for the new probes as well (Fig. 4). Other calibration procedures using network analyzers seem to have encountered similar interference, in one case at much higher frequency.

Fig. 4 shows a typical plot of $|\frac{V_{\text{meas}}}{V_{\text{ref}}}|$, the gain function of the probe, as measured through the differential amplifier, illustrating the dips that constitute the interference. Eq. 38 predicts an initially linear increase in the gain corresponding to the ideal case, Faraday’s Law. The leveling-off of the curve as the frequency increases is due to the increasing impedance of the probe’s self-inductance. The dips represent some other frequency-dependent reduction in the gain. Fig. 5 shows the best theoretical fit to the gain using Eq. 38, up to 60 MHz for a single axis.
Numerical integration was noted to be a potential source of error in data collection. The integration compounds the measurement error at each step resulting in drift of the computed magnetic field away from the actual field. Signal components close to the sampling frequency of the digitizer can cause poor aliasing which also results in drift of the numerical integration. A small offset bias in the measured signal can have the same effect as well.

Elimination of these error sources was the motivation of the present work. Experiments were conducted to determine and remove the source of the inference for this setup. An analog integrator was constructed and tested as a way to eliminate numerical integration error from the data analysis. The probes and the integrator were tested by measuring the magnetic field of a laser plasma.

IV. B-DOT CALIBRATION REFINEMENT

It was concluded that electromagnetic interference from the magnetic source was not the cause of the observed dips, since the stray signal produced by positioning the wire segment B-field source at the midpoint of the alumina shaft or the steel shaft was not great enough in amplitude to cause such dips. Resonance of the inductance of the coil and the capacitance of the twisted leads was another possible cause. However, the required capacitance value calculated to create a resonance at the observed frequencies, given the known inductance of the probe coils ($\sim 2.5 \times 10^{-7}$ H), was unrealistically large (hundreds of nanofarads).

In the case of a B-dot probe, an impedance mismatch in the coaxial cables could cause the measured signal amplitude to be reduced. Transmission line effects have been mentioned as a cause for non-ideal B-dot probe behavior.\(^1,11,14,16,18\) Among other things, transmission lines such as coaxial cables have a characteristic impedance $Z_0$ that is constant for all frequencies and cable lengths. Devices connected to the cable must have the same impedance as the cable; otherwise, a signal traveling through the cable will be partially reflected at the mismatch. This will result in standing waves or signal echoes in the cable.\(^19\)

Evidence clearly suggests that the interference observed in the current calibration method resulted from standing waves in the coaxial cables. First, the dips in the plots of the gain were found at frequencies following a linear relation, as can be seen in Fig. 6, which shows the frequency plotted against the number of dips. That the frequency is not in a direct proportion to the number of dips rules out the possibility this being an $LC$ resonance.
Rather, this pattern is almost certainly produced by resonant waves in the cable.

Second, it was discovered that the frequencies of the interference dips were dependent on the configuration of the coaxial cabling between the probe and the amplifier. Two different sets of coaxial cables were used, and in some tests the external conductors of the cables were grounded at the rear of the probe shaft by grounding the LEMO connectors (see Fig. 1). The results can be seen in Fig. 6, which shows the number of dips plotted against frequency, for the four possible cable configurations.

The two cable sets were of slightly different length, resulting in differing resonance frequencies when the LEMO connectors were not grounded (that is, were floating). However, the resonance frequencies were the same for the two cable sets when the LEMO connectors were grounded. This implies that the resonance now is independent of the length of the external cables. Thus when the LEMO connectors are grounded, the resonance must have been confined to the internal cables of the probe only. This is supported by the fact that the dips in the grounded condition are spaced further apart in frequency, corresponding to a shorter resonant wavelength. Thus, the resonances occur on the cable between the probe tip and the grounding point (which is naturally at the amplifier if the LEMO connectors are not also grounded).

The presence of resonance in the cables implies that impedance mismatches were present. An experimental probe was built with the inclusion of a 47 Ω resistor in series with each coil winding in an attempt to match the impedance of the probe tip to that of the coaxial cable. The resistance of a single coil was found to be 3-5 Ω, whereas the cable had an impedance
of 50 Ω. Using the inductance calculated from the calibration, the impedance of the probe should be less than 60 Ω up to 100 MHz. The calibration test showed reduced resonance at low frequency (Fig. 7). However, the probe still retained interference characteristic of resonance effects.

It was subsequently discovered that the twisted leads of the probe tip might act as a transmission line, with a characteristic impedance different from that of the coaxial cable. The characteristic impedance was estimated to be 100 Ω, so it is possible that the mismatch between this transmission line and the coaxial cable is one of the causes of the interference. However, the length of the leads is much shorter than the expected wavelengths involved, which implies that the twisted leads should behave as simple conductors rather than as a transmission line.

The analysis of the calibration would be simpler without the need to account for the interaction of the two loops and the behavior of the amplifier. Therefore an investigation was conducted into the behavior of a single loop circuit, independent of the differential amplifier. The signal from an individual loop was found to have significantly more noise than the combined signal from the differential amplifier. It seems that the differential amplifier was successfully removing a significant capacitive pickup from the field generator. However, it is unfortunate that the complication of the differential amplification scheme cannot be removed from the calibration of the probes.

The common mode rejection of the differential amplifier was found to be poor. A BNC
FIG. 8: Transmission of common mode through differential amplifier

splitter was used to send the same signal from the output of the network analyzer to each input of the differential amplifier. The resulting signal from the amplifier increased linearly from zero at low frequency, reaching $70 - 90\%$ transmission by $100 \text{ MHz}$. Fig. 8 shows the transmission fraction as a function of frequency for one of the three channels of the amplifier. The poor common mode rejection seems to contradict the conclusion reached above, that the amplifier successfully mitigates a significant amount of capacitive pickup. A better differential amplifier may be called for if high frequency capacitive pickup is expected from the plasma.

Although the calibration data was smoother with the outer conductors of the cables grounded, they were left floating for the experiment, since the cable behavior was not fully understood at the time. Similarly, the impedance-matched probe was not considered suitably characterized for use in the measurement of the laser plasma. Future work should ensure that all the elements of the probe are impedance-matched to the coaxial cable used, and that grounding is treated appropriately, as it may influence the probe behavior.

As previously mentioned, it was an objective of this work to produce a suitable analog integrator for fast B-dot signals, to replace the existing numerical integration. The circuit design shown in Fig. 9 was selected. It functions on the same principle as a classic Miller integrator\textsuperscript{19} and has approximately the same response function (Eq. 41).

$$V_{out}(t) = -\frac{1}{RC} \int V_{in}(t) dt \quad (41)$$

It differs from the basic integrator in that a potentiometer is used to produce a variable voltage reference point to compensate for DC offsets in the input signal. The integration
of an uncompensated, constant offset voltage would yield a linear voltage ramp in time, resulting in saturation of the integrator.

V. INTEGRATOR

The THS 4275 operational amplifier (op amp) was selected for the circuit, but a prototype was tested using a LM741 op amp with a bandwidth of 1 MHz. This prototype integrator failed at 1 MHz, indicating that this circuit is limited by the bandwidth of the op amp. The integrator (built as designed with the THS 4275 op amp) was shown to operate well at frequencies as low as 1 kHz using an oscilloscope and signal generator. Fig. 10 shows the response of the integrator to a 1 kHz square wave: the triangle wave is the integral of the square wave. No problems with saturation were encountered.

The higher frequency range of the integrator was tested using the network analyzer.
order to interpret the network analyzer output, we must use the frequency-domain form of
the ideal integrator equation. To do this, once again assume a complex exponential function
for the input voltage as in Eq. 22,

\[ V_{in} = V_0 e^{j\omega t}. \]

Then we substitute this formula into Eq. 41 to obtain the desired relation,

\[ V_{out}(\omega) = -\frac{1}{RC} \int V_{in}(\omega)dt \]  
\[ = -\frac{1}{RC} \int V_0 e^{j\omega t}dt \]  
\[ = -\frac{1}{RC} \left( \frac{1}{i\omega} V_0 e^{j\omega t} \right) \]  
\[ = \frac{i}{\omega RC} V_{in}(\omega). \]

This shows that a constant phase shift of \( i \), or 90°, should be produced, and that the
gain is inversely proportional to frequency. Fig. 11 shows the gain and Fig. 12 shows the phase difference for the integrator. Note that the phase is initially $90^\circ$ as desired, however due to cable length differences a constant time delay is introduced, which appears as a constant slope in the phase difference. The gain curve is linear with slope $-1$ in the log-log plot, indicating that it is inversely proportional to frequency, as desired. The phase plot demonstrates that the integrator functioned as designed up to $150 \, MHz$.

In testing, the baseline noise signal from the integrator was found to be up to three times larger than that of the differential amplifier. Additionally, the single laser plasma measurement taken with the integrator did not reveal any signal above this noise. It was subsequently discovered that this integrator design was optimized for lower frequencies, meaning that its gain was too low for the frequencies of interest. Further work should focus on finding the correct gain and on reducing noise by using better components and circuit board techniques.

VI. MEASUREMENT OF THE LASER PLASMA

As a test of the functionality of the standard B-dot probe, one such probe was used to measure the magnetic field of a laser plasma. The spontaneous magnetic field of such a plasma has been studied previously, so it was possible to compare the results obtained with prior work.\textsuperscript{10,12,21}

A. Experimental Arrangement

The tests were conducted of the main target chamber for the Phoenix Laser at UCLA. Figs. 13 and 14 show the probe and target inside the chamber. The Phoenix laser was fired at graphite rod, producing a plasma jet that was directed toward the B-dot probe.

The Phoenix Laser is a long pulse (5 ns FWHM), high power Nd:glass laser. It can deliver $30 \, J$ circularly polarized pulse on target with an intensity of up to $10^{14} W/cm^2$. It has energy stability of better than 10% and can fire once every three minutes.

For the present experiment the laser struck a graphite target with a spot size $\sim 350\mu m$, for an estimated peak intensity $10^{12} W/cm^2$. This should produce a plasma with an initial temperature of approximately $100 \, eV$ decreasing to $10 \, eV$ as the plasma expands (here
1 eV is equivalent to ∼ 1 million C°). The laser delivered 6-8 J to the target during this experiment. The beam struck the graphite at an angle of about 45° from the surface normal. The graphite target was 1 cm square and 6 cm tall, and was electrically isolated from the grounded chamber. The chamber was evacuated to $5 \times 10^{-5}$ Torr by a turbo-pump. The optical setup dictated that the probe axis be misaligned by about 10° from the direction of the plasma flow. Thus, varying the probe’s distance from the target also resulted in lateral displacement of the probe relative to the plasma, complicating data interpretation.

The data from the probe was recorded by a digital oscilloscope (Tektronix Digital Oscilloscope 7254). An ICCD gated camera (Princeton Instruments PI-Max) was used with a CIII filter to image the laser plasma from above at various times during the plasma expansion.
FIG. 15: Time evolution of plasma, from successive laser shots. Target is in top right corner, B-dot probe is the diagonal line. Note that the probe was translated between images.

(Fig. 15). The images show that the probe was indeed in the path of the plasma.

B. Results and Analysis

The ICCD images (Fig. 15) afford a rough initial velocity for the plasma of 240 km/s, which is in agreement with other laser plasma velocities. Fig. 17 shows a raw B-dot signal.
Notice the initial high amplitude, high frequency noise that is followed by the magnetic signal and the lower-frequency, low-amplitude noise. Fig. 17 shows the magnetic field derived from the numerical integration and correction of the B-dot signal. Notice the noise amplitude is much lower following the integration.

The magnetic field was found to always be strongest in the \(-y\) direction (see axis in Fig. 14). Since the probe tip was \(\sim 2\ cm\) in the \(-z\) direction (probe coordinates) from the centerline of the plasma jet, the probe’s \(+y\)-axis is equivalent to the \(−θ\) direction in cylindrical coordinates where \(+z\) is the target normal vector, pointing in the direction of the plasma velocity. Such a field corresponds to a net current in the \(-z\) direction (in the cylindrical coordinate frame), or into the target, as observed by others\(^3,10,12,21\). Some experiments have reported strong fields in the axial direction, caused by other source terms such as the dynamo term\(^{23}\). However, the axial field observed in this experiment is consistent
with zero, within experimental error. Apparently the axial source terms become insignificant far from the target and late in time, and the axial field decays.

The time delay between the laser firing and the peak magnetic field strength was determined and plotted against the distance of the probe from the target (Fig. 19). The slope of a linear fit to the data yields a speed of 140 km/s. This differs by a factor of about two from the estimated 230 km/s initial plasma speed. Bird et al predict and observe that the magnetic field motion is dominated by convection, rather than diffusion, for laser plasma conditions. However, Nakano & Sekiguchi observed two distinct radial regimes, composed of an inner plasma volume where the high temperature ensures that convection is dominant, and an outer region where the cool plasma allows diffusion to dominate. Thus, the magnetic field peak is found to be delayed in this outer region. It seems probable that in the present experiment the probe is situated in the cooler outer plasma, delaying the arrival of the peak field and resulting in a velocity lower than that of the plasma.

The peak-to-peak field strength (Fig. 18) seems to decrease linearly with increasing distance from the target; however, the last two points break this trend. This seeming discrepancy actually correlates well with the results observed in Fig. 3 in McKee et al. In this figure, the contour plot shows that the peak magnetic field strength at a given radius $r$ attains a maximum value at an axial distance comparable to $r$.

A reversed field is observed after the peak field has passed. Limited data from prior publications is available on the shape of the magnetic probe signals for comparison. The available plots indicate a magnetic field which does not reverse direction, having only a single peak. However, some simulations and experiments do show that field reversal
FIG. 19: Peak field arrival time versus distance

...can occur.\textsuperscript{3,7,10} According to McKee et al.,\textsuperscript{10} the reversal of the field late in time and well behind the plasma front is the result of $+r$ direction density gradient and $+z$ temperature gradient. Field reversal has been attributed to the presence of significant ambient plasma (produced by ionization of ambient gas by UV radiation from the laser plasma).\textsuperscript{3,10} However, the high vacuum of the present experiment should preclude the influence of background gas on the plasma, at least up to moderate distances from the target.\textsuperscript{3,10} Also, the reversed field predicted would occur most strongly at the front of the plasma, but the observed field reversal in this experiment occurs behind the main front. Therefore, the reversed field observed appears to be unrelated to the presence of any ambient plasma.

C. Evaluation of the Electronics

The numerical integration is relatively stable, especially when the amplitude of the actual B-dot signal is large. For traces with low B-dot signal (peak fields < 5 G), the noise and integration error prevented accurate field readings. In most other cases the integrated magnetic field returned to a value close to zero, showing that the cumulative integration error for these magnetic field traces was small. While the voltage resolution is small (8-bit), the high sampling rate of the Tektronix oscilloscope (2.5 GS/s on each channel) properly resolves the signal and the numerical integration error is generally low. However, even for some large-amplitude B-dot signals the integration does not return to zero. This makes a good case for the adoption of analog integration.
At early times electronic noise occurred in all the signals for which the laser beam struck the target, but did not occur when the laser was fired into a beam dump. The laser beam incident on the target apparently caused an electromagnetic pulse that resulted in ringing in some electronic components involved. The oscillations began simultaneously with the beams arrival, indicating that the plasma was not responsible for the pickup. Early in time, oscillations of 70-90 MHz occurred, but these died quickly, yielding an oscillation at 13 MHz. These oscillations can be seen in Fig. 16.

The signal from the differential amplifier showed evidence of a beat frequency. For one shot, the differential amplifier was bypassed and the signals from individual loops were recorded. The high frequency oscillations of two signals are initially in phase, and of the same polarity, but with slightly different frequencies. As they shifted out of phase, the differential amplifier could not cancel them. The only way to clean up the signal is to prevent these oscillations from occurring. To avoid capacitive pickup, some probe tips have been enclosed by grounded metal strips or foil, acting as Faraday cages. However, the mechanism by which the laser pulse excited these oscillations is unknown, so shielding may not be effective. The fact that the interference observed happens at higher frequency than the magnetic signal is beneficial. When integrated, the amplitude of the oscillations is lower. (Recall from the integrator discussion that the gain of the integration function is inversely proportional to the signal frequency.)

VII. SUMMARY

B-dot probes were constructed and calibrated following Everson et al. Interference occurring in the calibration process was traced to signal reflection and resonance. Impedance matching of the probe was shown to reduce this interference. Future work should begin by ensuring that proper grounding and impedance matching are applied to all elements of the probe circuit. An analog integrator was constructed and characterized. With some refinement, this design should prove suitable for integrating fast probe signals. Future work should focus on reducing noise, and on finding an appropriate level of gain. Finally, a probe was tested in a laser plasma and shown to provide useful data which compared well with previous experimental work. The numerical integration routine functions properly for most data sets. Stray oscillations occurred in the cabling. These oscillations were caused by a common
mode voltage, indicating that the differential amplifier is not always sufficient to eliminate electrostatic effects. Improved shielding or grounding of the probe may be necessary. This work should prove valuable by increasing the accuracy of magnetic field measurements in fast plasma phenomena, especially laser plasma.

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